

Hawking-Page phase transition of black Dp-branes and R-charged black holes with an IR cutoff

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ABSTRACT: We show that the confinement-deconfinement phase transition of supersymmetric Yang-Mills theories with 16 supercharges in various dimensions can be realized through the Hawking-Page phase transition between the near horizon geometries of black Dp-branes and BPS Dp-branes by removing a small radius region in the geometry in order to realize a confinement phase, which generalizes Herzog's discussion for the holographic hard-wall AdS/QCD model. Removing a small radius region in the gravitational dual corresponds to introducing an IR cutoff in the dual field theory. We also discuss the Hawking-Page phase transition between thermal AdS_5 , AdS_4 , AdS_7 spaces and R-charged AdS black holes coming from the spherical reduction of the decoupling limit of rotating D3-, M2-, and M5- branes in type IIB supergravity and 11 dimensional supergravity in grand canonical ensembles, where the IR cutoff also plays a crucial role in the existence of the phase transition.

KEYWORDS: AdS-CFT Correspondence, D-branes, Black Holes in String Theory.

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1. Introduction

The AdS/CFT correspondence [1–4] conjectures that type IIB string theory on $AdS_5 \times S^5$ is dual to $\mathcal{N} = 4$ $SU(N)$ supersymmetric Yang-Mills (SYM) theory in 3+1 dimensions. At low energies, the string theory can be approximated by supergravity on AdS_5 , while the SYM theory is a conformal field theory on the boundary of AdS_5 . At finite temperature, Witten related the Hawking-Page phase transition of black holes in AdS_5 space with the confinement-deconfinement phase transition of dual SYM [5]. On the gravity side, there are two classical solutions with the same boundary: the thermal AdS space and the Schwarzschild-AdS black hole which approaches AdS_5 asymptotically. As noted first by Hawking and Page [6], a first order phase transition occurs at some critical temperature, above which an AdS black hole forms. On the other hand, at a lower temperature, the thermal gas in AdS_5 dominates. This Hawking-Page phase transition is identified with the first order confinement-deconfinement phase transition of dual SYM theory: at low

temperature, the field theory is in a confinement phase and above a critical temperature it is in a deconfinement phase.

In Witten's example, the boundary on which the finite temperature field theory lives is a compact space $S^1 \times S^3$. The radius of the 3 dimensional sphere breaks the conformal symmetry of the field theory, which makes the phase transition possible. For the case with a non-compact boundary $S^1 \times R^3$, because of the conformal invariance, no Hawking-Page phase transition exists and on the SYM side only the deconfinement phase is present even in a finite temperature case [7–9]. However, the authors of ref. [10] are able to realize confinement in certain supersymmetric theories by removing a small radius part of the AdS geometry when the boundary is noncompact. In the framework of gauge/gravity correspondence, the radial coordinate on the gravity side corresponds to the energy scale on the field theory side. Thus the small radius cutoff on the gravity side implies introducing an IR cutoff in the field theory. The so-called hard wall AdS/QCD model has been extensively employed in discussing various properties of low energy QCD [11–20].

Then there is one point to remind here that for supersymmetric field theories which live on a non-compact space, introducing an IR cutoff is an effective way to realize a confinement-deconfinement phase transition, while for those which live on a flat but at least one dimension compact space, ie. $S^1 \times T^3$ or so, there is a kind of AdS soliton [21] which can be used to realize confinement. Hawking-Page phase transitions can occur between Ricci flat AdS black holes and AdS solitons both with at least one dimension compact, see, for example, [22–24].

AdS/CFT correspondence was first noticed by Maldacena when studying the decoupling limit of N coincident D3-branes. In the case of coincident D p -branes ($p \neq 3$), there are also correspondences of this kind between certain supergravity solutions and $SU(N)$ supersymmetric field theories with sixteen supercharges in $p + 1$ dimensions [25]. In the decoupling limit, the geometry of supergravity solutions is no longer AdS and in these cases the field theories are no longer conformal field theories. Although so, as in the case of D3-branes, the Hawking-Page phase transition does not happen when the boundary is noncompact, implying these field theories are in the deconfinement phase. In this paper we will study the confinement-deconfinement phase transition of these field theories by introducing an IR cutoff in the dual supergravity descriptions, which generalizes Herzog's discussion on the deconfinement transition of hard wall AdS/QCD model [11]. In the decoupling limit of rotating black D3-branes, M2-branes, and M5-branes, there also exist correspondences between R-charged AdS black holes and R-charged supersymmetric field theories at finite temperature [26–32]. In this paper, we will also study the confinement-deconfinement phase transition of these R-charged supersymmetric field theories with an IR cutoff in the dual description. Recently, the author of [33] studied the phase transition of AdS R-charged black holes. However the black holes discussed there are R-charged AdS black holes with spherical horizons; while we study the R-charged AdS black holes with Ricci flat horizons, which come from the sphere reduction of the decoupling limit of rotating black D3-, M2-, M5-branes.

The paper is organized as follows. In the next section, as a warmup exercise, we will briefly review the Hawking-Page phase transition for AdS black holes with the boundary

$S^1 \times R^3$. Then in section 3, we will study the Hawking-Page phase transition for the general case of near horizon limit of N coincident black Dp-branes, whose boundaries are non-compact $S^1 \times R^p$. In section 4, we study the case of the R-charged AdS_5 , AdS_4 , and AdS_7 black holes, respectively. Section 5 is devoted to conclusions.

2. Hawking-Page phase transition for Ricci flat black holes with an IR cutoff

In this section we review the deconfinement transition of hard-wall AdS/QCD through the Hawking-Page phase transition between thermal AdS_5 and an AdS_5 black hole with a non-compact boundary $S^1 \times R^3$. For more details, see [11]. In order to study the phase transition of the boundary CFT using the gravity description, we should first find the classical solutions of AdS supegravity with the same asymptotic boundary $S^1 \times R^3$, and then compare the Euclidean actions of these classical solutions to see if there is a phase transition. However, as we know, the actions always diverge due to the infinite space. There are two ways to get a finite result: one is to add surface counterterms to the action and the other is the so-called background subtraction method where a suitable reference background is chosen so that the solution under study can be asymptotically embedded into this background. Here we use the background subtraction method as it is more suitable to our purpose to calculate the difference of two Euclidean actions in this paper.

In the Euclidean sector, the action of 5-dimensional gravity with a cosmological constant can be written as

$$I = -\frac{1}{16\pi G} \int d^5x \sqrt{g}(R - 2\Lambda), \tag{2.1}$$

where Λ is the cosmological constant which can be related to the radius scale l of AdS space by $\Lambda = -6/l^2$. According to the action (2.1), there are two solutions with the same asymptotic boundary $S^1 \times R^3$, i.e. thermal AdS space and the AdS black hole solution (in Lorentz sector):

$$ds_{AdS}^2 = \frac{U^2}{l^2} (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{l^2}{U^2} dU^2, \tag{2.2}$$

$$ds_{BH}^2 = \frac{U^2}{l^2} \left[-\left(1 - \frac{U_H^4}{U^4}\right) dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \right] + \frac{l^2}{U^2} \left(1 - \frac{U_H^4}{U^4}\right)^{-1} dU^2, \tag{2.3}$$

where U_H corresponds to the horizon of the black hole. After a Euclidean continuation $t = i\tau$ the two solutions become

$$ds_{AdS}^2 = \frac{U^2}{l^2} (d\tau^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{l^2}{U^2} dU^2, \tag{2.4}$$

$$ds_{BH}^2 = \frac{U^2}{l^2} \left[\left(1 - \frac{U_H^4}{U^4}\right) d\tau^2 + dx_1^2 + dx_2^2 + dx_3^2 \right] + \frac{l^2}{U^2} \left(1 - \frac{U_H^4}{U^4}\right)^{-1} dU^2. \tag{2.5}$$

To eliminate the conical singularity, the τ in the AdS black hole solution should get a period

$$\beta = \frac{\pi l^2}{U_H}, \tag{2.6}$$

while the period of τ for the thermal AdS could be arbitrary. This period (2.6) is just the inverse of the temperature of the AdS black hole. To see whether there is a phase transition between the AdS black hole and thermal AdS space, we should calculate the difference of the Euclidean actions for these two solutions. The Euclidean actions of the AdS black hole and the thermal AdS are

$$I_{BH} = \frac{8}{16\pi G l^5} \int_{U_H}^{U_{uv}} d^5 x U^3, \tag{2.7}$$

$$I_{AdS} = \frac{8}{16\pi G l^5} \int_0^{U_{uv}} d^5 x U^3, \tag{2.8}$$

respectively. Here to get the actions of both solutions, we have introduced a finite UV boundary at $U = U_{uv}$. At the end of calculations, the limit $U_{uv} \rightarrow \infty$ will be taken. At the boundary, the temperatures for both solutions should be the same. This means that we have the following relation for the two temperatures

$$\beta_{AdS} = \beta \sqrt{1 - \frac{U_H^4}{U_{uv}^4}}. \tag{2.9}$$

It turns out that the difference of the two actions is

$$\Delta I = \lim_{U_{uv} \rightarrow \infty} (I_{BH} - I_{AdS}) = -\frac{V(\vec{x}) U_H^4 \beta}{16\pi G l^5} < 0, \tag{2.10}$$

where $V(\vec{x})$ denotes the volume of the three flat dimensions x_1, x_2 and x_3 . This negative action difference means that the black hole always dominates and confirms that the dual field theory is in the deconfinement phase. Now we introduce an IR cutoff U_{IR} in the coordinates (2.2), where the IR cutoff U_{IR} is equivalent to an IR cutoff (mass gap) in the dual field theory, then the integral in the action of thermal AdS should start from U_{IR} and the integral of the black hole geometry should start from $U_{\max} = \max[U_{IR}, U_H]$ [11, 18]. Now the Euclidean actions of the two solutions are

$$I_{BH} = \frac{V(\vec{x})\beta}{16\pi G l^5} (U_{uv}^4 - U_{\max}^4), \tag{2.11}$$

and

$$I_{AdS} = \frac{V(\vec{x})\beta_{AdS}}{16\pi G l^5} (U_{uv}^4 - U_{IR}^4), \tag{2.12}$$

respectively. Thus one has the action difference

$$\Delta I = \lim_{U_{uv} \rightarrow \infty} = \frac{V(\vec{x})\beta}{16\pi G l^5} \left(\frac{1}{2} U_H^4 - U_{\max}^4 + U_{IR}^4 \right). \tag{2.13}$$

The action difference obviously depends on the IR cutoff and U_{\max} . When the temperature is very low, U_H is small compared to U_{IR} , one then has $U_{\max} = U_{IR}$, and

$$\Delta I = \frac{V(\vec{x})\beta}{16\pi G l^5} \frac{1}{2} U_H^4 > 0. \tag{2.14}$$

On the other hand, when the temperature gets higher, U_H will become larger than U_{IR} , one takes $U_{\max} = U_H$, and has

$$\Delta I = \frac{V(\vec{x})\beta}{16\pi G l^5} \left(U_{IR}^4 - \frac{1}{2} U_H^4 \right). \quad (2.15)$$

Eq. (2.14) tells us that in the low temperature phase, where $U_{IR} > U_H$, the thermal gas in AdS dominates and there is no Hawking-Page transition; and it implies that the dual field theory is in the confinement phase. However, when $U_{IR} < U_H$, the action difference (2.15) can change its sign from positive to negative at a critical temperature where $U_{IR}^4 = \frac{1}{2} U_H^4$. The critical temperature is

$$\beta_{\text{crit}} = \frac{\pi l^2}{2^{\frac{1}{4}} U_{IR}}. \quad (2.16)$$

The Hawking-Page transition indicates that when $T > 1/\beta_{\text{crit}}$, the AdS black hole dominates, while the thermal AdS space dominates when $T < 1/\beta_{\text{crit}}$. In the dual field theory side, the field theory is in the deconfinement phase at $T > 1/\beta_{\text{crit}}$, while it is in the confinement phase at $T < 1/\beta_{\text{crit}}$. When the temperature T crosses the critical temperature $1/\beta_{\text{crit}}$, the deconfinement phase transition happens. The IR cutoff U_{IR} can be related to the mass of the lightest meson in the holographic AdS/QCD model [11]. As a result, we see that an IR cutoff can realize the Hawking-Page transition for Ricci flat AdS black holes when the boundary is non-compact. It is easy to understand the occurrence of the transition because an IR cutoff breaks the conformal symmetry for the dual field theory. Finally, we mention here that usually the Gibbons-Hawking surface term should be included in calculating the Euclidean action of black holes. However, for asymptotically AdS spacetimes it turns out that the surface term will not make a contribution to the action difference [6], which will be clearly seen in the next section.

3. Hawking-Page phase transition in black Dp-branes with IR cutoff

Like the D3-branes, the decoupling limit of Dp-branes in type II supergravity has also field theory description; they are supersymmetric Yang-Mills theories with 16 supercharges in $p + 1$ dimensions [25]. In this section, we will generalize the discussions in section 2 to the cases of those finite temperature non-conformal field theories defined on the boundary $S^1 \times R^p$ by studying the decoupling limit of Dp-branes. Generally to get a well-defined decoupling limit, p should be limited the range $0 \leq p \leq 4$.

To see whether there will be a phase transition for dual field theories at finite temperature, we will first get the two classical Euclidean solutions with the same asymptotic boundary $S^1 \times R^p$, and then compare the Euclidean actions of the two solutions in both cases with and without an IR cutoff. The two classical solutions can be obtained by taking the decoupling limits of black Dp-branes and BPS Dp-branes.

3.1 The decoupling limit of black Dp-branes

Black Dp-branes are non-BPS solutions of ten dimensional Type II supergravities. The

bulk action of the supergravity is

$$S_{\text{str}} = -\frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g} \left[e^{-2\phi} (R + 4(\partial\phi)^2) - \frac{1}{2d!} F_d^2 \right] \quad (3.1)$$

in string frame, and

$$S_{\text{Ein}} = -\frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g} \left[R - \frac{1}{2} (\partial\phi)^2 - \frac{e^{-\alpha(d)\phi}}{2d!} F_d^2 \right] \quad (3.2)$$

in Einstein frame, where $d = p + 2$ in the case of the electric brane and $d = 8 - p$ of the magnetic brane. $\alpha(d)$ depends on the value of d : $\alpha(d) = \frac{d-5}{2}$. Since F_{8-p} and F_{p+2} both do not change under the frame transformation while the metric changes, the duality relation changes from $F_{8-p} = *F_{p+2}$ in string frame to $F_{8-p} = e^{-\alpha(p+2)\phi} * F_{p+2}$ in Einstein frame.

The solution for N coincident black Dp-branes is

$$ds_s^2 = H^{-\frac{1}{2}}(r) \left(-f(r) dt^2 + \sum_{i=1}^p (dx^i)^2 \right) + H^{\frac{1}{2}}(r) (f^{-1}(r) dr^2 + r^2 d\Omega_{8-p}^2), \quad (3.3)$$

$$e^\phi = g_s H^{\frac{3-p}{4}}, \quad (3.4)$$

$$A_{t1\dots p} = g_s^{-1} (1 - H^{-1}) \coth \beta. \quad (3.5)$$

in string frame, where

$$H(r) = 1 + \xi \frac{c_p g_s N \alpha'^{\frac{7-p}{2}}}{r^{7-p}} = 1 + \xi \frac{r_p^{7-p}}{r^{7-p}} = 1 + \sinh^2 \beta \frac{r_H^{7-p}}{r^{7-p}}, \quad (3.6)$$

$$c_p \equiv (2\sqrt{\pi})^{5-p} \Gamma \left[\frac{1}{2}(7-p) \right], \quad (3.7)$$

$$\xi = \tanh \beta = \sqrt{1 + \left(\frac{r_H^{7-p}}{2r_p^{7-p}} \right)^2} - \frac{r_H^{7-p}}{2r_p^{7-p}}, \quad (3.8)$$

$$f(r) = 1 - \frac{r_H^{7-p}}{r^{7-p}}. \quad (3.9)$$

To get BPS Dp-branes, one can simply set $r_H = 0$ and then $f = 1$.

We can get the decoupling limit of this solution keeping the energies fixed by changing the parameters to [25]

$$N g_{\text{YM}}^2 = N (2\pi)^{p-2} g_s \alpha'^{\frac{p-3}{2}} = \text{fixed}, \quad (3.10)$$

$$U = \frac{r}{\alpha'} = \text{fixed}, \quad (3.11)$$

and setting

$$\alpha' \rightarrow 0, \quad U_H = \frac{r_H}{\alpha'}, \quad d_p = c_p (2\pi)^{2-p}. \quad (3.12)$$

In this decoupling limit, the solution in Einstein frame becomes

$$ds_{\text{Ein}}^2 = \alpha'^{\frac{7-p}{4}} \left\{ \frac{U^{\frac{(7-p)^2}{8}}}{(g_{\text{YM}}^2 d_p N)^{\frac{7-p}{8}}} \left[- \left(1 - \frac{U^{\frac{7-p}{H}}}{U^{7-p}} \right) dt^2 + d\vec{x}^2 \right] + \frac{(g_{\text{YM}}^2 d_p N)^{\frac{p+1}{8}}}{U^{\frac{(p+1)(7-p)}{8}}} \left[\frac{dU^2}{1 - \frac{U^{\frac{7-p}{H}}}{U^{7-p}}} + U^2 d\Omega_{8-p}^2 \right] \right\}, \quad (3.13)$$

$$e^\phi = \alpha'^{\frac{p-3}{2}} \left(\frac{g_{\text{YM}}^2 d_p N}{U^{7-p}} \right), \quad (3.14)$$

$$F_{U01\dots p} = \alpha'^{\frac{p+1}{2}} \frac{(p-7)(2\pi)^{p-2} U^{6-p}}{d_p N g_{\text{YM}}^4}. \quad (3.15)$$

This solution is just the gravitational dual of SYM theory with 16 supercharges in $p+1$ dimensions. When $p=3$, the solution turns out to be $AdS_5 \times S^5$, the dual theory is the $\mathcal{N}=4$ SYM theory with 32 charges. In that case, the theory is a conformal one. This case is just discussed in the previous section. Now we study the general cases without the conformal symmetry.

The Euclidean sector of the above solution can be obtained by setting $t = i\tau$

$$ds_{\text{Euc}}^2 = \alpha'^{\frac{7-p}{4}} \left\{ \frac{U^{\frac{(7-p)^2}{8}}}{(g_{\text{YM}}^2 d_p N)^{\frac{7-p}{8}}} \left[\left(1 - \frac{U^{\frac{7-p}{H}}}{U^{7-p}} \right) d\tau^2 + d\vec{x}^2 \right] + \frac{(g_{\text{YM}}^2 d_p N)^{\frac{p+1}{8}}}{U^{\frac{(p+1)(7-p)}{8}}} \left[\frac{dU^2}{1 - \frac{U^{\frac{7-p}{H}}}{U^{7-p}}} + U^2 d\Omega_{8-p}^2 \right] \right\}. \quad (3.16)$$

The Euclidean time τ has a period

$$\beta = \frac{4\pi}{\sqrt{\partial_U g_{\tau\tau} \partial_U g_{UU}^{-1}}} \Big|_{U=U_H} = \frac{4\pi g_{\text{YM}} \sqrt{d_p N}}{(7-p) U_H^{\frac{5-p}{2}}}, \quad (3.17)$$

in order to remove the conical singularity. This is nothing but the inverse Hawking temperature of the black D $_p$ -branes in the decoupling limit.

3.2 Phase transition with an IR cutoff

To see whether there is a phase transition between the decoupling limits of black D $_p$ -branes and BPS D $_p$ -branes, or say, deconfinement transition of those SYM theories at finite temperature, we first calculate the on-shell action of those black D $_p$ -branes. To avoid the complex surface term in the action for D $_p$ -branes with electric charge, we consider black D $_p$ -branes with magnetic charge. The on-shell Euclidean action can be written out using the equation of motion

$$I = \frac{7-p}{4} \frac{g_s^2}{16\pi G_{10}} \int d^{10}x \sqrt{g} \frac{e^{-\alpha(8-p)\phi} F_{8-p}^2}{2(8-p)!}. \quad (3.18)$$

Note the relation

$$\frac{e^{-\alpha(8-p)\phi} F_{8-p}^2}{2(8-p)!} = \frac{e^{-\alpha(p+2)\phi} F_{p+2}^2}{2(p+2)!}, \quad (3.19)$$

and one has then the Euclidean action

$$I_{\text{bulk}} = \alpha'^{7-p} \frac{(7-p)^3}{8} \frac{V(\vec{x})V(\Omega_{8-p})\beta}{16\pi G_{10}} \int U^{6-p} dU, \quad (3.20)$$

where $V(\vec{x})$ is the volume of the p spatial dimensions and $V(\Omega_{8-p})$ is the volume of unit $8-p$ sphere. The factor of α'^{7-p} can be absorbed into the redefinition of the Newton constant $G_{10} = 8\pi^6 g_s^2 \alpha'^4 = \alpha'^{7-p} 8\pi^6 g_{\text{YM}}^4 / (2\pi)^{2p-4} \equiv \alpha'^{7-p} G'_{10}$ in the decoupling limit.

We first calculate the difference of the bulk actions of the decoupling limits of the black Dp-branes and BPS Dp-branes. To regularize the actions, we introduce a UV boundary U_{uv} for both solutions, where the local temperatures are the same for both solutions. Here we use I_{bl} as the Euclidean action of the decoupling limit of the black Dp-branes and I_{ba} as the Euclidean action of the decoupling limit of the BPS Dp-branes. Thus we have

$$I_{\text{bulk}}^{bl} = \frac{(7-p)^3}{8} \frac{V(\vec{x})V(\Omega_{8-p})\beta}{16\pi G'_{10}} \int_{U_H}^{U_{uv}} U^{6-p} dU, \quad (3.21)$$

and

$$I_{\text{bulk}}^{ba} = \frac{(7-p)^3}{8} \frac{V(\vec{x})V(\Omega_{8-p})\beta'}{16\pi G'_{10}} \int_0^{U_{uv}} U^{6-p} dU, \quad (3.22)$$

where

$$\beta' = \beta \sqrt{1 - \frac{U_H^{7-p}}{U_{uv}^{7-p}}}. \quad (3.23)$$

The difference of these two actions is

$$\begin{aligned} \Delta I_{\text{bulk}} &= \lim_{U_{uv} \rightarrow \infty} (I_{bl} - I_{ba}) \\ &= \lim_{U_{uv} \rightarrow \infty} \frac{(7-p)^2}{8} \frac{V_p V(\Omega_{8-p})\beta}{16\pi G'_{10}} \left[U_{uv}^{7-p} \left(1 - \sqrt{1 - \frac{U_H^{7-p}}{U_{uv}^{7-p}}} \right) - U_H^{7-p} \right] \\ &= \frac{(7-p)^2}{8} \frac{V_p V(\Omega_{8-p})\beta}{16\pi G'_{10}} \left(-\frac{1}{2} \right) U_H^{7-p} < 0. \end{aligned} \quad (3.24)$$

Besides the bulk part, we should also consider the contribution of the Gibbons-Hawking surface term

$$I_{GB} = -\frac{1}{8\pi G_{10}} \int_{\partial M} d^9 x \sqrt{h} K, \quad (3.25)$$

where h is the determinant of the reduced metric on the UV boundary ∂M and K is the extrinsic curvature of the reduced metric

$$K = \nabla_\mu n^\mu. \quad (3.26)$$

The surface terms for both solutions are

$$I_{GB}^{bl} = -\frac{V(\vec{x})V(\Omega_{8-p})\beta}{16\pi G'_{10}} \left[\left(16 - 2p - \frac{(7-p)(p+1)}{8} \right) U_{uv}^{7-p} - \left(9 - p - \frac{(7-p)(p+1)}{8} \right) U_H^{7-p} \right], \quad (3.27)$$

and

$$I_{GB}^{ba} = -\frac{V(\vec{x})V(\Omega_{8-p})\beta'}{16\pi G'_{10}} \left[\left(16 - 2p - \frac{(7-p)(p+1)}{8} \right) U_{uv}^{7-p} \right], \quad (3.28)$$

respectively. Then the difference of the two surface terms is

$$\Delta I_{GB} = \frac{V(\vec{x})V(\Omega_{8-p})\beta}{16\pi G'_{10}} \left(\frac{p-3}{4} \right)^2 U_H^{7-p}. \quad (3.29)$$

When $p = 3$, this term vanishes. This confirms that for AdS black holes, the Gibbons-Hawking surface term has no contribution to the Euclidean action difference stated in the previous section. Finally we get the total Euclidean action difference for those two solutions

$$\Delta I = \Delta I_{\text{bulk}} + \Delta I_{GB} = -\frac{V(\vec{x})V(\Omega_{8-p})\beta}{16\pi G'_{10}} \left(\frac{5-p}{2} \right) U_H^{7-p}. \quad (3.30)$$

Here we should note that we choose the total action to be the sum of the bulk term and the Gibbons-Hawking boundary term. There are no other surface counterterms like boundary cosmological counterterms used in [34] needed here because we are using the background subtraction method. And the result we get here is the same as in [34] where they also calculated the Euclidean action of the near horizon geometry of black Dp-branes (compactified on transverse spheres) but using the counterterm approach.

Then from the equation above we can see that this action difference is always negative and hence no Hawking-Page phase transition happens. This means that the near horizon geometries of black Dp-branes dominate all the times, and the dual field theories are always in the deconfinement phase. Now as in the hard-wall AdS/QCD model, we introduce an IR cutoff to realize a confinement phase. Correspondingly, we introduce an IR cutoff U_{IR} in the dual gravitational description by removing the part with $U < U_{IR}$ of geometry. With the IR cutoff, the integral in the action starts from U_{IR} in the case of the near horizon limit of Dp-branes and $U_{\text{max}} = \max[U_{IR}, U_H]$ in the case of the near horizon limit of black Dp-branes. In this case, the difference of the actions becomes

$$\begin{aligned} \Delta I_{\text{bulk}} &= \lim_{U_{uv} \rightarrow \infty} (I_{\text{bulk}}^{bl} - I_{\text{bulk}}^{ba}) \\ &= \frac{(7-p)^2}{8} \frac{V(\vec{x})V(\Omega_{8-p})\beta}{16\pi G'_{10}} \left(\frac{1}{2} U_H^{7-p} - U_{\text{max}}^{7-p} + U_{IR}^{7-p} \right), \end{aligned} \quad (3.31)$$

while the part from the Gibbons-Hawking surface term keeps unchanged, still has the form (3.29). Thus, we have

$$\begin{aligned} \Delta I &= \Delta I_{\text{bulk}} + \Delta I_{GB} \\ &= \frac{V(\vec{x})V(\Omega_{8-p})\beta}{16\pi G'_{10}} \left(\frac{p^2 - 10p + 29}{8} U_H^{7-p} - \frac{(7-p)^2}{8} (U_{\text{max}}^{7-p} - U_{IR}^{7-p}) \right). \end{aligned} \quad (3.32)$$

When $U_H < U_{IR}$, one has $U_{\max} = U_{IR}$, and

$$\Delta I = \frac{V(\vec{x})V(\Omega_{8-p})\beta}{16\pi G'_{10}} \left[\left(\frac{p^2 - 10p + 29}{8} \right) U_H^{7-p} \right] > 0. \quad (3.33)$$

On the other hand, when $U_H > U_{IR}$, we have $U_{\max} = U_H$, and

$$\Delta I = \frac{V(\vec{x})V(\Omega_{8-p})\beta}{16\pi G'_{10}} \left(-\frac{5-p}{2} U_H^{7-p} + \frac{(7-p)^2}{8} U_{IR}^{7-p} \right). \quad (3.34)$$

We see that the action can change its sign and the Hawking-Page phase transition happens when $U_H^{7-p} = \frac{(7-p)^2}{4(5-p)} U_{IR}^{7-p}$. The corresponding critical temperature is

$$\beta_{crit} = \frac{4\pi g_{YM} \sqrt{d_p N}}{(7-p) \left(\frac{(7-p)^2}{4(5-p)} U_{IR}^{7-p} \right)^{\frac{5-p}{2}}}. \quad (3.35)$$

Because the temperature of the black Dp-branes is proportional to $U_H^{(5-p)/2}$ (see (3.17)), it is easy to see that at low temperature less than $1/\beta_{crit}$, the decoupling limit of Dp-branes with IR cutoff dominates, which corresponds to the confinement phase of dual SYM theories; and at high temperature above the critical temperature, the decoupling limit of black Dp-branes dominates, which corresponds to the deconfinement phase of the dual SYM theories. In addition, we mention again that here p is in the range $0 \leq p \leq 4$.

Thus we have shown that as in the case of the hard-wall AdS/QCD model, one also can realize the deconfinement transition for $p+1$ dimensional SYM theories residing on non-compact manifold $S^1 \times R^p$ through the first order Hawking-Page phase transition between the decoupling limits of black Dp-branes and BPS Dp-branes by introducing an IR cutoff.

4. Hawking-page phase transition of R-charged AdS_4 , AdS_5 , and AdS_7 black holes with an IR cutoff

The decoupling limit of the solution of N coincident rotating black D3-branes of the ten dimensional type IIB supergravity action can be reduced to 5 dimensions through S^5 dimensional reduction, resulting in a 5 dimensional charged AdS black hole [29–32]. According to AdS/CFT correspondence, these charged AdS black holes in five dimensions are dual to R-charged SYM theory living on the boundary. Also the decoupling limits of the solutions of N coincident rotating black M2 and M5-branes of the 11 dimensional supergravity can be reduced to charged AdS_4 and AdS_7 black holes through S^7 and S^4 reductions respectively [29]. These R-charged AdS black holes are black holes with Ricci flat horizon. In this section we discuss the Hawking-Page phase transitions of those Ricci flat AdS black holes in grand canonical ensembles. Note that the Hawking-Page phase transition in those R-charged AdS black holes with spherical horizon has been discussed in [31, 32], while it has been studied for the case with hyperbolic horizon in [35].

4.1 R-charged AdS_5 black holes

In the case of rotating D3-branes, there are six spatial dimensions transverse to the branes, so there can be at most 3 angular momenta. Thus after dimensional reduction on S^5 , there can be three charges, parameterized by q_1, q_2 and q_3 respectively. The action after spherical reduction becomes [29]

$$I = -\frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R - \frac{1}{2}(\partial\vec{\varphi})^2 - \frac{1}{4} \sum_i X_i^2 (F_i)^2 + \frac{4}{l^2} \sum_i X_i^{-1} \right), \quad (4.1)$$

where

$$X_i = e^{-\frac{1}{2}\vec{a}_i \cdot \vec{\varphi}} \quad (4.2)$$

with dilation vectors

$$\vec{a}_1 = \left(\frac{2}{\sqrt{6}}, \sqrt{2} \right), \quad \vec{a}_2 = \left(\frac{2}{\sqrt{6}}, -\sqrt{2} \right), \quad \vec{a}_3 = \left(-\frac{4}{\sqrt{6}}, 0 \right). \quad (4.3)$$

This is just the action of a $U(1)^3$ truncation of the $\mathcal{N} = 8, SO(6)$ gauged supergravity. The solution after reduction is a black hole solution of this action with three charges under the $U(1)^3$ and two scalar fields. This solution is

$$ds^2 = -(\mathcal{H}_1\mathcal{H}_2\mathcal{H}_3)^{-\frac{2}{3}} f dt^2 + (\mathcal{H}_1\mathcal{H}_2\mathcal{H}_3)^{\frac{1}{3}} (f^{-1} dr^2 + r^2(dx_1^2 + dx_2^2 + dx_3^2)), \quad (4.4)$$

$$X_i = \mathcal{H}_i^{-1} (\mathcal{H}_1\mathcal{H}_2\mathcal{H}_3)^{\frac{1}{3}}, \quad A_i = \frac{\sqrt{\mu} (1 - \mathcal{H}_i^{-1})}{q_i}, \quad (4.5)$$

where

$$f = \frac{r^2}{l^2} \mathcal{H}_1\mathcal{H}_2\mathcal{H}_3 - \frac{\mu}{r^2}, \quad \mathcal{H}_i = 1 + \frac{q_i^2}{r^2}, \quad i = 1, 2, 3 \quad (4.6)$$

and μ is the mass parameter of the AdS black hole.

The black hole has the Hawking temperature $1/\beta$,

$$\beta = \left(\frac{4\pi}{(\mathcal{H}_1\mathcal{H}_2\mathcal{H}_3)^{-\frac{1}{2}} \partial_r f} \right) \Big|_{r=r_0}, \quad (4.7)$$

where r_0 corresponds to the black hole horizon, i.e., the largest real root of $f(r) = 0$,

$$\mu l^2 = r_0^4 \mathcal{H}_1(r_0) \mathcal{H}_2(r_0) \mathcal{H}_3(r_0). \quad (4.8)$$

And the Euclidean action becomes

$$I_E = -\frac{1}{16\pi G_5} \int d^5x \sqrt{g} \left(R - \frac{1}{2}(\partial\vec{\varphi})^2 - \frac{1}{4} \sum_i X_i^2 (F_i)^2 + \frac{4}{l^2} \sum_i X_i^{-1} \right). \quad (4.9)$$

4.1.1 Euclidean action for AdS_5 R-charged black holes

Before discussing the phase transition, we should state that we work in the grand canonical ensemble where the chemical potentials of the ensemble are fixed to certain values. The choice of ensemble is crucial because in grand canonical ensembles the Euclidean action can be just identified with the Gibbs free energy, while for canonical ensembles the Helmholtz free energy should be given by the Legendre transform of the Euclidean action. Thus here we just need to calculate the difference of Euclidean actions as before. Now the background we choose is still the pure thermal AdS_5 space with zero valued charges but constant and maybe nonzero chemical potentials. Then to discuss the Hawking-Page phase transition associated with the AdS_5 R-charged black holes, we first calculate the on-shell action by using the Einstein equation. The Euclidean action for this solution is

$$I_E = \frac{4}{3} \frac{V(\vec{x})}{16\pi G_5 l^2} \int d\tau dr r \left(6r^2 + 2(q_1^2 + q_2^2 + q_3^2) - \sum_i \frac{\mu l^2 q_i^2}{(r^2 + q_i^2)^2} \right). \quad (4.10)$$

We will study the phase transition in the grand canonical ensemble, where the electric potentials are fixed. We can choose a certain gauge here to make

$$A_i = \frac{\sqrt{\mu} (1 - \mathcal{H}_i^{-1})}{q_i} + \Phi_i = 0 \quad (4.11)$$

at the horizon $r = r_0$. The gauge invariant chemical potential between the black hole horizon and infinity is Φ_i , since only this quantity enters into the action and other physical quantities.

To calculate the Euclidean action difference, we have to select an appropriate background. For the case of R-charged black holes, it is natural to select the pure AdS space-time with constant chemical potentials Φ_i , since this background is still the solution of equations of motion. Next we have to fix the period of Euclidean time of the pure AdS space-time. This can be done by equating the induced metric of the pure AdS space-time on the hypersurface $r = \text{constant}$ with the one of black hole [36]. This means we have

$$\int d\tau d^3x \sqrt{h} = \int d\tau' d^3x \sqrt{h'}, \quad (4.12)$$

where h and h' are the determinants of the induced metrics of black hole and the pure AdS space-time. Thus we find

$$\beta' = \beta \frac{\int d^3x \sqrt{h}}{\int d^3x \sqrt{h'}}, \quad (4.13)$$

where the integration is taken on the $r = r_{uv}$ hypersurface (an UV boundary). For the 3-charged black hole, we have

$$\beta' = \beta \sqrt{\frac{l^2 (\mathcal{H}_1 \mathcal{H}_2 \mathcal{H}_3)^{\frac{1}{3}} f}{r^2}} \Big|_{r_{uv}}. \quad (4.14)$$

For convenience we write

$$\mathcal{H}_1 \mathcal{H}_2 \mathcal{H}_3 = \left(1 + \frac{q_1^2}{r^2}\right) \left(1 + \frac{q_2^2}{r^2}\right) \left(1 + \frac{q_3^2}{r^2}\right) = 1 + \frac{A}{r^2} + \frac{B}{r^4} + \frac{C}{r^6}, \quad (4.15)$$

with A, B, C defined as follows

$$A = q_1^2 + q_2^2 + q_3^2, \quad B = q_1^2 q_2^2 + q_1^2 q_3^2 + q_2^2 q_3^2, \quad C = q_1^2 q_2^2 q_3^2. \quad (4.16)$$

Let us first consider the case without an IR cutoff. In this case, the Euclidean action of the black hole solution is

$$I_{\text{bulk}}^{bl} = \frac{2V(\vec{x})\beta}{16\pi G_5 l^2} \left[(r_{uv}^4 - r_0^4) + \frac{2A}{3}(r_{uv}^2 - r_0^2) + \frac{\mu l^2}{3} \sum_i \left(\frac{q_i^2}{r_{uv}^2 + q_i^2} - \frac{q_i^2}{r_0^2 + q_i^2} \right) \right], \quad (4.17)$$

while for the pure AdS background, we have

$$I_{\text{bulk}}^{ba} = \frac{2V(\vec{x})\beta'}{16\pi G_5 l^2} (r_{uv}^4). \quad (4.18)$$

Here we should note that just as stated in the beginning of this subsection, we are working in grand canonical ensemble and the background thermal AdS_5 spacetime has nonzero constant chemical potentials as in reference [32], so the Euclidean action which only involves the gauge field strength but not the gauge potential is unaffected by the nonzero fixed potentials.

The action difference is

$$\Delta I_{\text{bulk}} = \frac{V(\vec{x})\beta}{16\pi G_5 l^2} \left(-r_0^4 - Ar_0^2 - \frac{C}{r_0^2} + \frac{1}{9}(2A^2 - 15B) \right). \quad (4.19)$$

The contribution of the Gibbons-Hawking surface term for the black hole solution is

$$\begin{aligned} I_{GH}^{bl} &= -\frac{1}{8\pi G_5} \int_{\partial M} d\tau d^3x \sqrt{h} K \\ &= \frac{V(\vec{x})\beta}{8\pi G_5 l^2} \left(4r_{uv}^4 + \frac{8}{3}Ar_{uv}^2 + \left(\frac{4}{3}B - 2\mu l^2 \right) + \mathcal{O}\left(\frac{1}{r_{uv}}\right) \right). \end{aligned} \quad (4.20)$$

For the pure AdS background it is

$$I_{GH}^{ba} = -\frac{1}{8\pi G_5} \int_{\partial M} d\tau d^3x \sqrt{h} K = \frac{V(\vec{x})\beta'}{8\pi G_5 l^2} (4r_{uv}^4). \quad (4.21)$$

As a result, the part of action difference from the Gibbons-Hawking surface term is

$$\Delta I_{GH} = \frac{V(\vec{x})\beta}{8\pi G_5 l^2} \left[\frac{4}{9}(A^2 - 3B) \right]. \quad (4.22)$$

Thus we get the total action difference between the black hole and pure AdS space

$$\Delta I_{\text{bulk}} + \Delta I_{GH} = \frac{V(\vec{x})\beta}{16\pi G_5 l^2} \left(-\mu l^2 - \frac{2}{3}(q_1^4 + q_2^4 + q_3^4 - q_1^2 q_2^2 - q_2^2 q_3^2 - q_1^2 q_3^2) \right). \quad (4.23)$$

The appearance of the non-linear terms of charges in this formula is due to the asymptotical behavior of the scalar fields. When $\mu = 0$, those terms do not vanish. This is not a reasonable result. As argued in ref. [46], we should add a counterterm $\int d\tau d^3x \sqrt{h} \phi^2$ to the

action, which just cancels the part $-\frac{2}{3}(q_1^4 + q_2^4 + q_3^4 - q_1^2 q_2^2 - q_2^2 q_3^2 - q_1^2 q_3^2)$. Finally we arrive at

$$\Delta I = \frac{V(\vec{x})\beta}{16\pi G_5 l^2} (-\mu l^2). \quad (4.24)$$

Thus we find that this action difference is always negative, which means that no Hawking-Page transition happens between the AdS_5 black hole and the thermal AdS_5 space-time here, and the dual R-charge field theories are always in the deconfinement phase.

It should be noted here, the counterterm $\int d\tau d^3x \sqrt{\bar{h}} \bar{\phi}^2$ in the gauged supergravity is just a special form of counterterms to eliminate the non-linear terms of charges and divergent terms. There are general counterterms for general gauged supergravity theories, which have been discussed in [47]. For this 5-dimensional R-charged Ricci flat AdS black hole, one can find this counterterm

$$\int d\tau d^3x \sqrt{\bar{h}} (W(\phi) - 3/l),$$

where $W(\phi)$ is superpotential, and we have subtracted the contribution of the gravity counterterm $\int d\tau d^3x \sqrt{\bar{h}} 3/l$. After substituting the explicit form of $W(\phi)$ given in [47], one finds the non-linear charge term is precisely cancelled. This counterterm is equivalent to the counterterm $\int d\tau d^3x \sqrt{\bar{h}} \bar{\phi}^2$. In fact, for some ϕ_0 we have $W(\phi_0) = 3/l$, so expand $W(\phi) - 3/l$ around ϕ_0 , and one can get expression like $\int d\tau d^3x \sqrt{\bar{h}} \bar{\phi}^2$. We will give more detail discussion for this counterterm in the next section.

Now we turn to the case with an IR cutoff r_{IR} . As in the case of Schwarzschild-AdS black holes, we introduce $r_{\max} = \max[r_0, r_{IR}]$. The integral of the background starts from r_{IR} to r_{uv} and the integral of the black hole starts from r_{\max} to r_{uv} . We obtain the total action difference after adding the counterterm

$$\Delta I = \frac{V(\vec{x})\beta}{16\pi G_5 l^2} \left(\mu l^2 + 2r_{IR}^4 - 2r_{\max}^4 - \frac{4}{3} A r_{\max}^2 - \frac{2}{3} B - \sum_i \frac{2}{3} \frac{\mu l^2 q_i^2}{r_{\max}^2 + q_i^2} \right). \quad (4.25)$$

When $r_0 < r_{IR}$, one has $r_{\max} = r_{IR}$, and

$$\Delta I = \frac{V(\vec{x})\beta}{16\pi G_5 l^2} \left(\mu l^2 - \frac{4}{3} A r_{IR}^2 - \frac{2}{3} B - \sum_i \frac{2}{3} \mu l^2 q_i^2 \frac{1}{r_{IR}^2 + q_i^2} \right). \quad (4.26)$$

On the other hand, when $r_0 > r_{IR}$, one obtains $r_{\max} = r_0$, and

$$\Delta I = \frac{V(\vec{x})\beta}{16\pi G_5 l^2} (2r_{IR}^4 - \mu l^2). \quad (4.27)$$

When $r_{IR} = 0$, the action difference reduces to the one (4.24) without the IR cutoff.

4.1.2 Phase transition with an IR cutoff

Here we will discuss the thermodynamics in grand canonical ensemble, where the chemical potentials and temperature are fixed parameters. The IR cutoff r_{IR} for this ensemble is a fixed but arbitrary constant. Since when r_0 is large enough μl^2 becomes very large and the action difference (4.27) becomes a large negative quantity, the de-confinement phase always

exists. Then as long as the confinement phase exists, a phase transition will happen. That means to realize a phase transition we only have to ensure a positive action difference in a certain region of the phase diagram. Here we give a careful analysis to see whether the IR cutoff really helps the phase transition, and if it does, what values should the IR cutoff takes.

Given some fixed q_i 's, there always exists a value of r_0 which is denoted by $r_{0c}(q_i)$ such that $2r_0^4 > \mu l^2$ if $r_0 > r_{0c}(q_i)$. This $r_{0c}(q_i)$ always exists because μl^2 approaches r_0^4 when r_0 is large enough. Thus we can always find an IR cutoff $r_{IR} > r_{0c}(q_i)$ which satisfies $2r_{IR}^4 - \mu l^2 > 0$ when $r_0 > r_{IR}$. This means that confinement phase always exists in the (r_0, q_i) -space after giving an appropriate IR cutoff, and this appropriate IR cutoff can always be found.

But we are more interested in whether a confinement phase exists in the (T, Φ_i) phase diagram, since the ensemble we are considering is the grand canonical one. Note that Φ_i 's depend on q_i 's and r_0 ,

$$(\Phi_i)^2 \propto \frac{q_i^2(r_0^2 + q_1^2)(r_0^2 + q_2^2)(r_0^2 + q_3^2)}{r_0^2(r_0^2 + q_i^2)^2}.$$

To keep Φ_i 's unchanged, the charge parameters q_i 's have to change simultaneously when r_0 changes. As $r_0 \rightarrow \infty$, q_i 's change slowly and tend to fixed values $q_i = const. \times \Phi_i$. In other words, fixed chemical potentials are equivalent to fixed charge parameters when r_0 approaches infinity. However, generally q_i 's have an evaluated region, which is denoted by \mathcal{Q} , when r_0 changes. This means a fixed chemical potential Φ_i corresponds to a set of q_i 's. Certainly any meaningful charge parameter q_i can not be infinity, so \mathcal{Q} is a bounded region. Now take r_{0c} to be $\max[r_{0c}(q_i), q_i \in \mathcal{Q}]$. Then from the discussion in the previous paragraph, we can always find an IR cutoff $r_{0c} < r_{IR} < r_0$ such that $2r_{IR}^4 - \mu l^2 > 0$. Thus for any fixed chemical potentials, the confinement phase always exists. Then we can get to a conclusion that the introduction of an IR cutoff can realize a positive action difference for a system with any values of chemical potentials if the value of the IR cutoff is chosen properly.

Thus we conclude that if the IR cutoff is chosen properly, we can get a positive action difference for the case $r_0 > r_{IR}$ (4.27) and then to realize a confinement phase. Then we can say: by introducing the IR cutoff, the action difference (4.27) can change its sign, and then the Hawking-Page transition can occur. It implies that the confinement-deconfinement transition can happen for the dual field theory. This means that as the case without charges, the IR cutoff leads to the existence of the confinement phase. When temperature is high enough, the deconfinement transitions happens. In this case, the critical temperature of transition for the deconfinement transition is

$$T_c = \frac{1}{\beta} = \frac{r_0^2 r_{IR}^4 + 2r_0^6 - 2(q_1^2 q_2^2 + q_1^2 q_3^2 + q_2^2 q_3^2) r_0^4 - 4q_1^2 q_2^2 q_3^2}{4\pi l^2 r_0^2 \sqrt{(r_0^2 + q_1^2)(r_0^2 + q_2^2)(r_0^2 + q_3^2)}}. \quad (4.28)$$

From this critical temperature we find that r_0 has a minimum to assure a positive temperature, but this does not matter the discussion above.

Since the analytic analysis is not easy to make, in what follows, we move on to show some phase diagrams in several cases. We should plot the phase diagrams with chemical

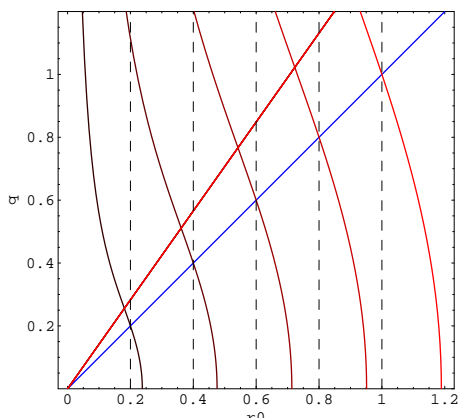


Figure 1: $r_0 - q$ phase diagram of 5-dimensional R-charged black holes with $q_1 = q, q_2 = q_3 = 0$. The solid curves correspond to the phase transition curves, and each curve has a fixed IR cutoff r_{IR} . With the colors changing from black to red, the values of r_{IR} increase from 0.2 to 1.0 with a step 0.2. The dashed curves stand for $r_0 = 0.2, 0.4, 0.6, 0.8, 1.0$, respectively. The straight blue curve describes the requirement of $r_0 > r_{IR}$, and the straight red curve divides the diagram into two parts by local stability of thermodynamics, below which the thermodynamics is local stable.

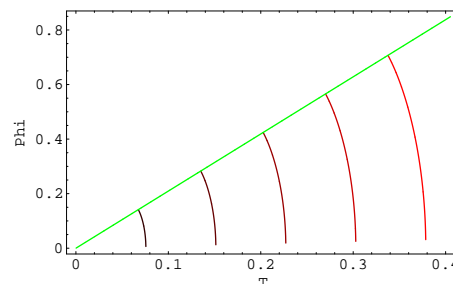


Figure 2: $T - \phi$ phase diagram of 5-dimensional R-charged black hole with $q_1 = q, q_2 = q_3 = 0$. The green curves correspond to the requirement $r_{IR} < r_0$, here only the stable part is shown. The solid curves correspond to the phase transition curves, and each curve has a fixed IR cutoff r_{IR} . With the colors changing from black to red, the values of r_{IR} increase from 0.2 to 1.0 with a step 0.2.

potentials and temperatures as variables and plot out the curve where the phase transition happens. Besides, we also plot out the phase diagrams in terms of charge parameters q and horizon radius r_0 . These two kinds of diagrams are equivalent after using the transformation relations.

Figure 1, 3 and 5 are $r_0 - q$ phase diagrams of the case $q_1 = q \neq 0, q_2 = q_3 = 0$, $q_1 = q_2 = q \neq 0, q_3 = 0$ and $q_1 = q_2 = q_3 = q \neq 0$, respectively. In these figures, the solid curves correspond to the phase transition curves, across which the action difference changes its sign, and each curve has a fixed IR cutoff r_{IR} . With the colors changing from black to red, the values of r_{IR} increase from 0.2 to 1.0 with a step 0.2. The dashed curves stand for $r_0 = 0.2, 0.4, 0.6, 0.8, 1.0$. This means that these straight blue curves describe the requirement of $r_0 > r_{IR}$. Thus, in fact, only the regions below these blue curves are meaningful for our discussion. From the $r_0 - q$ diagrams, one can read out the value of the IR cutoff by the intersecting points of the blue curves and the transition curves.

Figure 2, 4 and 6 are $T - \Phi$ diagrams for the case $\Phi_1 = \phi, \Phi_2 = \Phi_3 = 0, \Phi_1 = \Phi_2 = \phi, \Phi_3 = 0$, and $\Phi_1 = \Phi_2 = \Phi_3 = \phi$, respectively. In these figures, the green curves correspond to the requirement $r_{IR} < r_0$. With the color changing from black to red, the values of r_{IR} increase from 0.2 to 1.0 with a step 0.2. In the paper [18], the charged RN black holes discussed there are just R-charged AdS black holes with equal R charges. Here

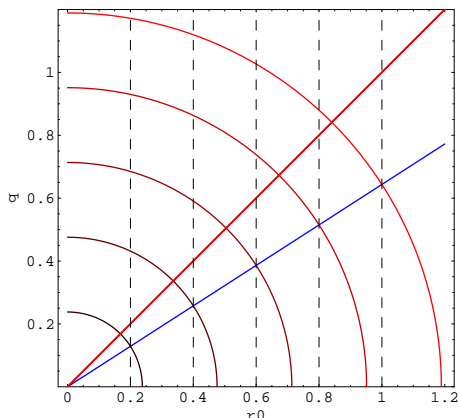


Figure 3: $r_0 - q$ phase diagram of 5-dimensional R-charged black hole with $q_1 = q_2 = q, q_3 = 0$.

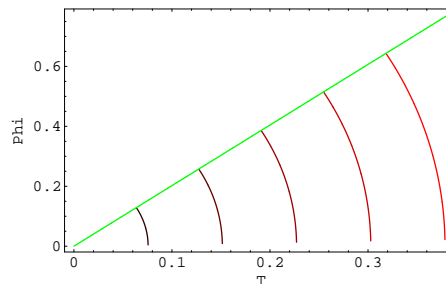


Figure 4: $T - \phi$ phase diagram of 5-dimensional R-charged black hole with $q_1 = q_2 = q, q_3 = 0$.

each $T - \phi$ phase diagram is plotted with 5 different values of r_{IR} to show its influence. The colors of the curves represent the values of r_{IR} , the darker, the smaller.

In the $r_0 - q$ diagrams we also plot the boundary for local thermodynamic stability [30, 31]. The straight red curves with $q = \sqrt{2}r_0$, $q = r_0$ and $q = r_0$ in these $r_0 - q$ diagrams correspond to the local thermodynamic stability curves. The local stability curves are determined by the Hessian of the Euclidean action

$$I = \beta(E - \Phi_i Q_i) - S, \quad (4.29)$$

with respect to r_0 and charge parameters q_i 's keeping β and Φ_i 's fixed, where E is the mass, Q_i 's are physical charges and S is the entropy of the R-charged black holes. These thermodynamic quantities can be got from the general thermodynamic relations

$$E = \left(\frac{\partial I}{\partial \beta} \right)_{\Phi_i} - \frac{\Phi_i}{\beta} \left(\frac{\partial I}{\partial \Phi_i} \right)_{\beta}, \quad S = \beta \left(\frac{\partial I}{\partial \beta} \right)_{\Phi_i} - I, \quad Q_i = -\frac{1}{\beta} \left(\frac{\partial I}{\partial \Phi_i} \right)_{\beta}. \quad (4.30)$$

In this case, the energy will have a constant correction due to the IR cutoff, while the entropy and physical charges do not change,

$$\begin{aligned} E &= \frac{V(\vec{x})}{16\pi G_5 l^2} (3\mu l^2 + 2r_{IR}^4), \\ S &= \frac{V(\vec{x})}{4G_5} \sqrt{(r_0^2 + q_1^2)(r_0^2 + q_2^2)(r_0^2 + q_3^2)}, \\ Q_i &= \frac{V(\vec{x})}{8\pi G_5} \frac{q_i}{r_0} \sqrt{(r_0^2 + q_1^2)(r_0^2 + q_2^2)(r_0^2 + q_3^2)}. \end{aligned} \quad (4.31)$$

The similar form of these quantities can be found in [31, 24]. The local stability curves are represented by the red straight curves under which the thermodynamics is locally stable. In addition, let us mention that in the $q - r_0$ phase diagrams, only the regions under the blue curves are physically allowed when the IR cutoff is introduced, since we are considering the

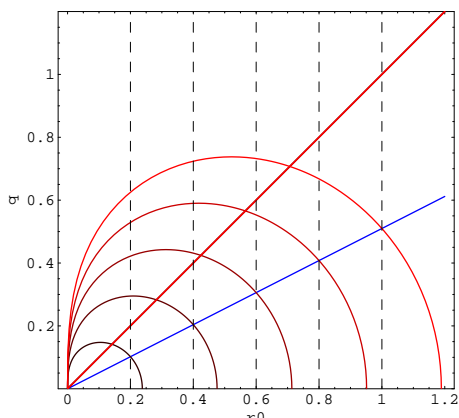


Figure 5: $q - r_0$ phase diagram for the 5-dimensional R-charged black hole with $q_1 = q_2 = q_3 = q$.

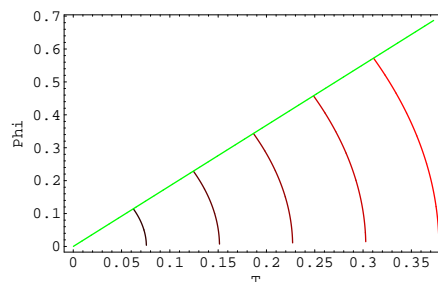


Figure 6: $\Phi - T$ phase diagram for the 5-dimensional R-charged black hole with $q_1 = q_2 = q_3 = q$.

case with $r_0 > r_{IR}$. As a result, we can see from these diagrams that the deconfinement phase transition always exists and the IR cutoff will not affect the local thermodynamical stability of the field theories. In the $T - \phi$ phase diagrams we only plot out the regions, corresponding to the ones under the blue curves of the $r_0 - q$ diagrams.

4.2 R-charged AdS_4 black holes

For rotating M2-branes in 11 dimensional supergravity, there are 8 transverse spatial dimensions. Thus there can be at most 4 angular momenta. After dimensional reduction, there will be at most 4 charges parameterized by $q_i, i = 1, 2, 3, 4$.

The decoupling limit of the rotating black M2-brane after reduction is four dimensional AdS black hole, which can be written as [29]

$$ds^2 = -(\mathcal{H}_1\mathcal{H}_2\mathcal{H}_3\mathcal{H}_4)^{-1/2} f dt^2 + (\mathcal{H}_1\mathcal{H}_2\mathcal{H}_3\mathcal{H}_4)^{1/2} (f^{-1} dr^2 + r^2(dx_1^2 + dx_2^2)) \quad (4.32)$$

where

$$f = -\frac{\mu}{r} + \frac{4r^2}{l^2} \mathcal{H}_1\mathcal{H}_2\mathcal{H}_3\mathcal{H}_4, \quad \mathcal{H}_i = 1 + \frac{\mu \sinh^2 \beta_i}{r}, \quad (4.33)$$

$$X_i = \mathcal{H}_i^{-1} (\mathcal{H}_1\mathcal{H}_2\mathcal{H}_3\mathcal{H}_4)^{1/4}, \quad A_t^i = \frac{1 - \mathcal{H}_i^{-1}}{\sinh \beta_i}. \quad (4.34)$$

Define $q_i^2 = \mu \sinh^2 \beta_i$, then we have

$$\mathcal{H}_i = 1 + \frac{q_i^2}{r}, \quad A_t^i = \frac{\sqrt{\mu}(1 - \mathcal{H}_i^{-1})}{q_i}. \quad (4.35)$$

The effective action in 4 dimensions is

$$I = -\frac{1}{16\pi G_4} \int \sqrt{-g} \left(R - \frac{1}{2} (\partial\vec{\varphi})^2 + \frac{4}{l^2} \sum_{i<j} X_i X_j - \frac{1}{4} \sum_i X_i^{-2} (F^i)^2 \right), \quad (4.36)$$

The relation between scalars X_i and $\vec{\varphi} = (\varphi_1, \varphi_2, \varphi_3)$ is given by

$$X_i = e^{-\frac{1}{2}\vec{a}_i \cdot \vec{\varphi}}, \quad (4.37)$$

where the vectors \vec{a}_i are given by

$$\vec{a}_1 = (1, 1, 1), \quad \vec{a}_2 = (1, -1, -1), \quad \vec{a}_3 = (-1, 1, -1), \quad \vec{a}_4 = (-1, -1, 1). \quad (4.38)$$

4.2.1 Euclidian action of AdS_4 R-charged black holes

Here and in the next section we also work in the grand canonical ensemble where the chemical potentials are fixed at the boundary as in the case of AdS_5 R-charged black holes, and the reference background is also pure thermal AdS_4 spacetime with zero valued charges but maybe nonzero electric potentials. Then we come to the calculation of the difference of Euclidean actions of the two solutions.

Substituting the black hole solution into the action, we get the on-shell Euclidean action

$$I = \frac{V(\vec{x})}{16\pi G_4} \int d\tau dr \left[-\frac{\mu}{2} \sum_{i=1}^4 \frac{q_i^2}{(r + q_i^2)^2} + \frac{4}{l^2} (6r^2 + 3Ar + B) \right]. \quad (4.39)$$

Here we have introduced the following quantities

$$A = \sum_i q_i^2, \quad B = \sum_{i < j} q_i^2 q_j^2, \quad C = \sum_{i < j < k} q_i^2 q_j^2 q_k^2, \quad D = q_1^2 q_2^2 q_3^2 q_4^2. \quad (4.40)$$

We first consider the case without an IR cutoff. In this case, the bulk action for the black hole is

$$I_{\text{bulk}}^{bl} = \frac{V(\vec{x})\beta}{16\pi G_4} \left[\frac{\mu}{2} \sum_{i=1}^4 \left(\frac{q_i^2}{r_{uv} + q_i^2} - \frac{q_i^2}{r_0 + q_i^2} \right) + \frac{8}{l^2} (r_{uv}^3 - r_0^3) \right. \\ \left. + \frac{6}{l^2} A (r_{uv}^2 - r_0^2) + \frac{4}{l^2} B (r_{uv} - r_0) \right], \quad (4.41)$$

and for the pure AdS_4 , the bulk action is

$$I_{\text{bulk}}^{ba} = \frac{V(\vec{x})\beta'}{16\pi G_4} (8r_{uv}^3), \quad (4.42)$$

which is also unaffected by the values of electric potentials by the same reason argued in the case of AdS_5 R-charged black holes. The contributions from the Gibbons-Hawking surface term are

$$I_{GH}^{bl} = -\frac{V(\vec{x})\beta}{8\pi G_4 l^2} \left(12r_{uv}^3 + 9Ar_{uv}^2 + 6Br_{uv} + 3C - \frac{3l^2}{2}\mu + \dots \right), \quad (4.43)$$

and

$$I_{GH}^{ba} = -\frac{1}{8\pi G_4} \int d\tau d^2x \sqrt{h} K = -\frac{V(\vec{x})\beta'}{8\pi G_4 l^2} (12r_{uv}^3), \quad (4.44)$$

respectively, for the black hole solution and pure AdS space, where from the equation (4.13), we have the relation between the two temperatures

$$\beta' = \beta \sqrt{\frac{(\mathcal{H}_1 \mathcal{H}_2 \mathcal{H}_3 \mathcal{H}_4)^{\frac{1}{2}} f l^2}{4r^2}} \Big|_{r_{uv}}, \quad (4.45)$$

where β is the inverse Hawking temperature of the black hole. Because there is something subtle in this case, we write out the relation between β' and β explicitly as follows,

$$\beta r_{uv}^3 - \beta' r_{uv}^3 = \beta \left(-\frac{3}{4} A r_{uv}^2 + \frac{3}{32} (A^2 - 8B) r_{uv} + \frac{\mu l^2}{8} + s_1(q_1, q_2, q_3, q_4) \right), \quad (4.46)$$

where there is a non-linear charge term which can be written as

$$\begin{aligned} s_1(q_1, q_2, q_3, q_4) = \frac{1}{128} [& -5(q_1^6 + q_2^6 + q_3^6 + q_4^6) + 9(q_1^4 q_2^2 + q_1^4 q_3^2 + q_1^4 q_4^2 \\ & + q_2^4 q_1^2 + q_2^4 q_3^2 + q_2^4 q_4^2 + q_3^4 q_1^2 + q_3^4 q_2^2 + q_3^4 q_4^2 + q_4^4 q_1^2 \\ & + q_4^4 q_2^2 + q_4^4 q_3^2) - 54(q_1^2 q_2^2 q_3^2 + q_1^2 q_2^2 q_4^2 + q_1^2 q_3^2 q_4^2 + q_2^2 q_3^2 q_4^2)]. \end{aligned} \quad (4.47)$$

Then we find

$$\begin{aligned} \Delta I_{\text{total}} = \lim_{r_{uv} \rightarrow \infty} \frac{V(\vec{x})\beta}{16\pi G_4} \left[-\frac{\mu}{2} \sum_{i=1}^4 \left(\frac{q_i^2}{r_0 + q_i^2} \right) - \frac{1}{2l^2} (3A^2 - 8B) r_{uv} \right. \\ \left. - 16 \frac{s_1}{l^2} - 6 \frac{C}{l^2} + \mu - 8 \frac{r_0^3}{l^2} - 6 \frac{A r_0^2}{l^2} - 4 \frac{B r_0}{l^2} + \dots \right]. \end{aligned} \quad (4.48)$$

Note that r_0 is the horizon of the black hole, satisfying

$$-\frac{\mu}{r_0} + \frac{4r_0^2}{l^2} \mathcal{H}_1(r_0) \mathcal{H}_2(r_0) \mathcal{H}_3(r_0) \mathcal{H}_4(r_0) = 0, \quad (4.49)$$

we arrive at

$$\Delta I_{\text{total}} = \lim_{r_{uv} \rightarrow \infty} \frac{V(\vec{x})\beta}{16\pi G_4} \left[-\mu - \frac{1}{2l^2} (3A^2 - 8B) r_{uv} - 16 \frac{s_1}{l^2} - 4 \frac{C}{l^2} \right]. \quad (4.50)$$

One can see that as $r_{uv} \rightarrow \infty$, the action diverges, unless the four charges are equal or at least equal two by two. This problem is common in theories with scalar fields. The divergence is due to the asymptotical behavior of these scalar fields. The similar problem arises in the so called ‘‘boundary counterterm’’ method [37–41, 46]. In these references, one can remove the divergence by adding a counterterm I_{ct}^g into the action,

$$I = I_{\text{bulk}} + I_{GH} + I_{ct}^g. \quad (4.51)$$

These counterterms are constructed by boundary curvature,

$$\begin{aligned} I_{ct}^g = \frac{1}{8\pi G_d} \int d^{d-1} x \sqrt{h} \left[(d-2)/l + \frac{l}{2(d-3)} \mathcal{R} \right. \\ \left. + \frac{l^3}{2(d-3)^2(d-5)} \left(\mathcal{R}^{ab} \mathcal{R}_{ab} - \frac{d-1}{4(d-2)} \mathcal{R}^2 \right) + \dots \right], \end{aligned} \quad (4.52)$$

where $\mathcal{R}, \mathcal{R}_{ab}$ are Ricci scalar and Ricci tensor of the boundary. Thus one can call them gravity counterterms, and denote the sum by an index g . However, for theories with scalar fields, the divergence can not be eliminated even after one has added the gravity counterterm. To eliminate the divergence, generalized counterterms for the theories with scalars should be added as follows.

$$I_{ct} = \frac{1}{8\pi G_d} \int d^{d-1}x \sqrt{h} \left[W(\phi) + \frac{l}{2(d-3)} \mathcal{R} + \frac{l^3}{2(d-3)^2(d-5)} \left(\mathcal{R}^{ab} \mathcal{R}_{ab} - \frac{d-1}{4(d-2)} \mathcal{R}^2 \right) + \dots \right], \quad (4.53)$$

where $W(\phi)$ is the superpotential and $\mathcal{R}, \mathcal{R}_{ab}$ are the Ricci scalar and Ricci tensor of the boundary.

This kind of counterterm was first derived in [42] for the domain wall solution in five dimensional supergravity, and the subsequent [43] for a more complete derivation. Here, the superpotential counterterm is by no means the only one needed. In general one also needs counterterms involving derivatives of scalars. By using Hamiltonian/Hamilton-Jacobi methods, the general analysis for gravity coupled to scalars with the complete set of counterterms has been given [44]. And more information about the counterterm of the system with scalar fields coupling to gravity can be found in [40, 44, 45, 47].¹

Here, since we are interested in the cases of Ricci flat black holes, this boundary counterterm is fully determined by the superpotential

$$I_{ct} = \frac{1}{8\pi G_d} \int d^{d-1}x \sqrt{h} W(\phi), \quad (4.54)$$

for any dimension. Certainly, with this counterterm, one can give appropriate Euclidean action for the black holes without considering the procedure of selecting a proper background. However, here since we are discussing the possible Hawking-Page phase transitions between the black hole and the background spacetime, it is more natural to use the background subtraction method. Thus in what follows we will subtract the contribution of the pure gravity counterterm, which means that the counterterm should be

$$I_{ct}^s = \frac{1}{8\pi G_d} \int d^{d-1}x \sqrt{h} (W(\phi) - (d-2)/l). \quad (4.55)$$

For the $D = 4$ R-charged black hole, we have (noting the AdS scale l in the function f of (4.33) is different from the standard one by a factor “1/2”, so in the following calculation we have to change l in (4.55) to be $l/2$)

$$W(\phi) = \frac{1}{l} \sum_i X_i, \quad X_i = e^{-\frac{1}{2} \vec{a}_i \cdot \vec{\phi}} \quad (4.56)$$

Thus, the counterterm for this four dimensional R-charged black hole becomes

$$I_{ct}^s = \frac{1}{8\pi G_4 l} \int d\tau d^2x \sqrt{h} \left(\sum_i X_i - 4 \right). \quad (4.57)$$

¹We would like thank Kostas Skenderis for useful comments on this point.

It is easy to find that the integrand in the above equation has the following expansion

$$\frac{1}{l}\sqrt{h}\left(\sum_i X_i - 4\right) = \frac{1}{4l^2}(3A^2 - 8B)r_{uv} - \frac{1}{16l^2}s_2(q_1, q_2, q_3, q_4) + \dots, \quad (4.58)$$

where there are non-linear charge terms like the one in (4.50), which is denoted by

$$\begin{aligned} s_2(q_1, q_2, q_3, q_4) = & 5(q_1^6 + q_2^6 + q_3^6 + q_4^6) - 9(q_1^4 q_2^2 + q_1^4 q_3^2 + q_1^4 q_4^2 \\ & + q_2^4 q_1^2 + q_2^4 q_3^2 + q_2^4 q_4^2 + q_3^4 q_1^2 + q_3^4 q_2^2 + q_3^4 q_4^2 + q_4^4 q_1^2 \\ & + q_4^4 q_2^2 + q_4^4 q_3^2) + 22(q_1^2 q_2^2 q_3^2 + q_1^2 q_2^2 q_4^2 + q_1^2 q_3^2 q_4^2 + q_2^2 q_3^2 q_4^2). \end{aligned} \quad (4.59)$$

Note that the first term in (4.58) precisely cancels the divergence term in the action difference (4.50), while the second term in (4.58) exactly remove the non-linear charge terms by following relation

$$16s_1 + 4C = -\frac{1}{8}s_2, \quad (4.60)$$

so after considering this counterterm, we finally get the Euclidean action difference between the black hole and pure AdS background

$$\Delta I = -\frac{V(\vec{x})\beta}{16\pi G_4}\mu < 0, \quad (4.61)$$

which means that there is no phase transition in this case, and the black hole solution dominates and the dual field theory is in the deconfinement phase.

Now we turn to the case with an IR cutoff r_{IR} . In this case the contributions from the Gibbons-Hawking surface term and the counterterm which are calculated on the UV boundary are not affected, and the bulk part changes to

$$\begin{aligned} I_{\text{bulk}}^{bl} = & \frac{V(\vec{x})\beta}{16\pi G_4} \left[\frac{\mu}{2} \sum_{i=1}^4 \left(\frac{q_i^2}{r_{uv} + q_i^2} - \frac{q_i^2}{r_{\text{max}} + q_i^2} \right) + \frac{8}{l^2}(r_{uv}^3 - r_{\text{max}}^3) \right. \\ & \left. + \frac{6}{l^2}A(r_{uv}^2 - r_{\text{max}}^2) + \frac{4}{l^2}B(r_{uv} - r_{\text{max}}) \right], \end{aligned} \quad (4.62)$$

where we have introduced $r_{\text{max}} = \max[r_0, r_{IR}]$. The action of the background becomes

$$I_{\text{bulk}}^{ba} = \frac{V(\vec{x})\beta'}{16\pi G_4 l^2}(8r_{uv}^3 - 8r_{IR}^3). \quad (4.63)$$

Considering the contributions from the Gibbons-Hawking surface terms and counterterms, we obtain the total action difference

$$\begin{aligned} \Delta I = & \frac{V(\vec{x})\beta}{16\pi G_4} \left(-\mu + \frac{8}{l^2}r_{IR}^3 + \frac{\mu}{2} \sum_{i=1}^4 \left(\frac{q_i^2}{r_0 + q_i^2} \right) - \frac{\mu}{2} \sum_{i=1}^4 \left(\frac{q_i^2}{r_{\text{max}} + q_i^2} \right) \right. \\ & \left. + \frac{8}{l^2}r_0^3 - \frac{8}{l^2}r_{\text{max}}^3 + \frac{6}{l^2}Ar_0^2 - \frac{6}{l^2}Ar_{\text{max}}^2 + \frac{4}{l^2}Br_0 - \frac{4}{l^2}Br_{\text{max}} \right). \end{aligned} \quad (4.64)$$

When $r_0 < r_{IR}$, one should have $r_{\max} = r_{IR}$, and

$$\Delta I = \frac{V(\vec{x})\beta}{16\pi G_4} \left(-\mu + \frac{8}{l^2} r_{IR}^3 + \frac{\mu}{2} \sum_{i=1}^4 \left(\frac{q_i^2}{r_0 + q_i^2} \right) - \frac{\mu}{2} \sum_{i=1}^4 \left(\frac{q_i^2}{r_{IR} + q_i^2} \right) + \frac{8}{l^2} r_0^3 - \frac{8}{l^2} r_{IR}^3 + \frac{6}{l^2} A r_0^2 - \frac{6}{l^2} A r_{IR}^2 + \frac{4}{l^2} B r_0 - \frac{4}{l^2} B r_{IR} \right). \quad (4.65)$$

On the other hand, when $r_0 > r_{IR}$, we obtain $r_{\max} = r_0$. Considering (4.49), we have a simple expression for the action difference

$$\Delta I = \frac{V(\vec{x})\beta}{16\pi G_4} \left(-\mu + \frac{8}{l^2} r_{IR}^3 \right). \quad (4.66)$$

This is just the one we want. When $r_{IR} \rightarrow 0$, the action reduces to the case without an IR cutoff.

4.2.2 Phase transition with an IR cutoff

Next we analyze the phase structure for the case with $r_0 > r_{IR}$. From equation (4.49) we find that

$$\mu = \frac{4}{l^2} r_0^3 \mathcal{H}_1(r_0) \mathcal{H}_2(r_0) \mathcal{H}_3(r_0) \mathcal{H}_4(r_0)$$

approaches to $\frac{4}{l^2} r_0^3$ when r_0 is large enough. Thus as argued in the five dimensional case, for any values of q_i 's, there always exists a value of r_0 which is denoted by $r_{0c}(q_i)$ such that $\frac{8}{l^2} r_0^3 > \mu$ if $r_0 > r_{0c}(q_i)$. This $r_{0c}(q_i)$ always exists due to the properties of μ . Therefore, we can always find an IR cutoff $r_{0c}(q_i) < r_{IR} < r_0$ which satisfies $\frac{8}{l^2} r_{IR}^3 - \mu > 0$. The latter indicates a confinement phase. This means the confinement phase always exists in the (r_0, q_i) -space once an appropriate IR cutoff is given.

On the other hand, when $\mu > \frac{8}{l^2} r_{IR}^3$, the action difference turns to be negative. In this case, the black hole solution dominates and the dual field theory is in the deconfinement phase. Therefore when μ crosses $\frac{8}{l^2} r_{IR}^3$, the Hawking-Page (deconfinement) phase transition happens.

Figure 7, 9, 11 and 13 plot the $r_0 - q$ phase diagrams for the case $q_1 = q, q_2 = q_3 = q_4 = 0$, $q_1 = q_2 = q, q_3 = q_4 = 0$, $q_1 = q_2 = q_3 = q, q_4 = 0$, and $q_1 = q_2 = q_3 = q_4 = q$, respectively. The five solid curves correspond to the phase transition curves, and each curve has a fixed IR cutoff r_{IR} . With the colors changing from black to red, the values of r_{IR} increase from 0.2 to 1.0 with a step 0.2. The dashed curves stand for $r_0 = 0.2, 0.4, 0.6, 0.8$, and 1.0, respectively. The blue curves represent the requirement of $r_0 > r_{IR}$. In these figures, the red curves which start from the origin correspond to $2q^2 = 3r_0$, $q^2 = r_0$, $q^2 = r_0$ and $q^2 = r_0$, respectively. The thermodynamics is local stable in the region under those red curves. These curves are determined by the Hessian of the Euclidean action with respect to r_0 and q_i with β and Φ_i fixed. Since the regions below these blue curves satisfy the requirement with $r_0 > r_{IR}$, therefore those regions are always local thermodynamical stable.

Figure 8, 10, 12 and 14 plot the $T - \phi$ phase diagrams for the case of $\Phi_1 = \phi, \Phi_2 = \Phi_3 = \Phi_4 = 0$, $\Phi_1 = \Phi_2 = \phi, \Phi_3 = \Phi_4 = 0$, $\Phi_1 = \Phi_2 = \Phi_3 = \phi, \Phi_4 = 0$, and $\Phi_1 = \Phi_2 =$

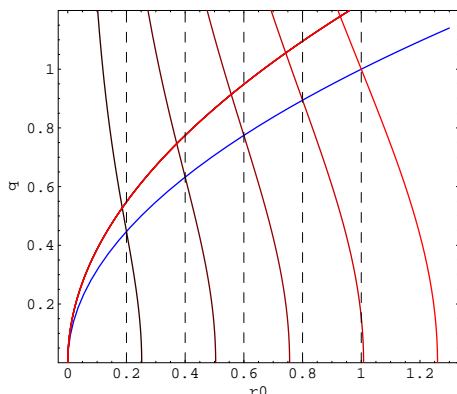


Figure 7: $r_0 - q$ phase diagram of 4-dimensional R-charged black hole with $q_1 = q, q_2 = q_3 = q_4 = 0$.

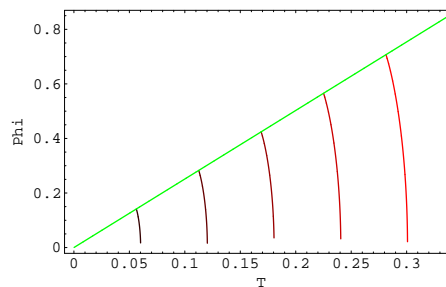


Figure 8: $T - \phi$ phase diagram of 4-dimensional R-charged black hole with $q_1 = q, q_2 = q_3 = q_4 = 0$.

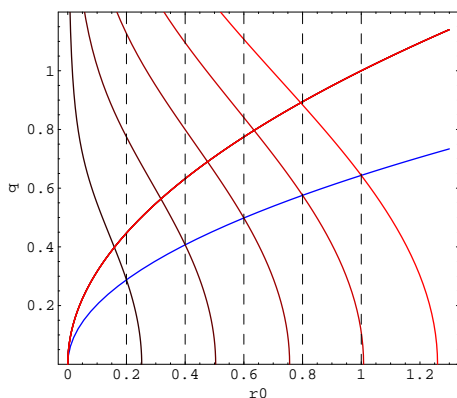


Figure 9: $r_0 - q$ phase diagram of 4-dimensional R-charged black hole with $q_1 = q_2 = q, q_3 = q_4 = 0$.

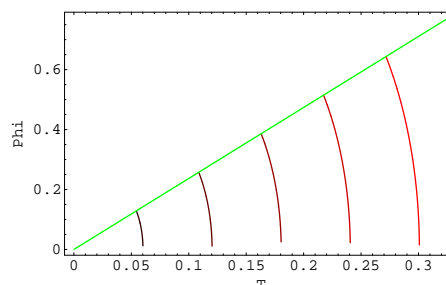


Figure 10: $T - \phi$ phase diagram of 4-dimensional R-charged black hole with $q_1 = q_2 = q, q_3 = q_4 = 0$.

$\Phi_3 = \Phi_4 = \phi$, respectively. The green curves correspond to the requirement $r_{IR} < r_0$. With the color changing from black to blue, the values of r_{IR} increase from 0.2 to 1.0 with a step 0.2. Again, we only plot the region satisfying the requirement $r_{IR} < r_0$.

4.3 R-charged AdS_7 black holes

The R-charged AdS_7 black holes have at most two charges parameterized by q_1 and q_2 . The solution can be written out by dimensional reduction from 11 dimensional rotating black M5 branes under decoupling limit [29]

$$\begin{aligned}
 ds_7^2 &= -(\mathcal{H}_1\mathcal{H}_2)^{-\frac{4}{5}} f dt^2 + (\mathcal{H}_1\mathcal{H}_2)^{\frac{1}{5}} (f^{-1} dr^2 + r^2 d\vec{x}^2), \\
 X_i &= \mathcal{H}_i^{-1} (\mathcal{H}_1\mathcal{H}_2)^{\frac{2}{5}}, \\
 f &= \frac{r^2}{4l^2} \mathcal{H}_1\mathcal{H}_2 - \frac{\mu}{r^4}, \\
 A_t^i &= \frac{\sqrt{\mu}(1 - \mathcal{H}_i^{-1})}{4lq_i}.
 \end{aligned} \tag{4.67}$$

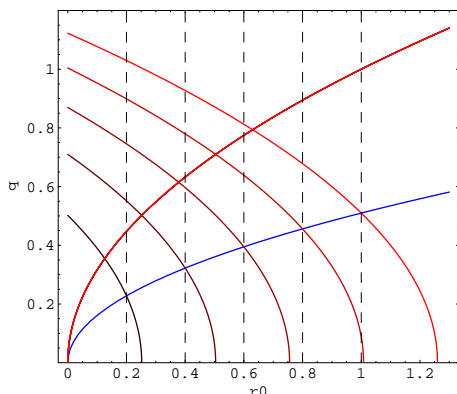


Figure 11: $r_0 - q$ phase diagram of 4-dimensional R-charged black hole with $q_1 = q_2 = q_3 = q, q_4 = 0$.

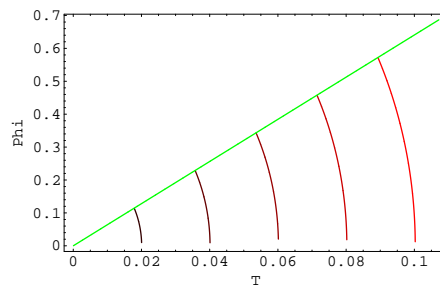


Figure 12: $T - \phi$ phase diagram of 4-dimensional R-charged black hole with $q_1 = q_2 = q_3 = q, q_4 = 0$.

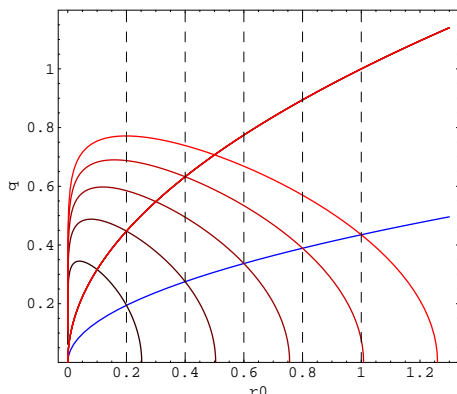


Figure 13: $r_0 - q$ phase diagram of 4-dimensional R-charged black hole with $q_1 = q_2 = q_3 = q_4 = q$.

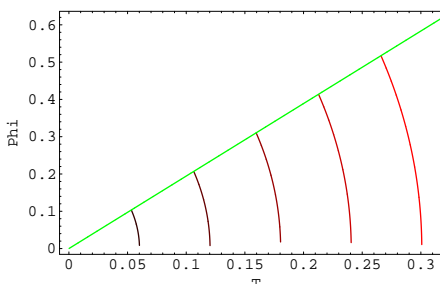


Figure 14: $T - \phi$ phase diagram of 4-dimensional R-charged black hole with $q_1 = q_2 = q_3 = q_4 = q$.

The effective action in 7 dimensions is

$$I = -\frac{1}{16\pi G_7} \int d^7x \sqrt{-g} \left(R - \frac{1}{2}(\partial\vec{\varphi})^2 - \frac{V}{l^2} - \frac{1}{4} \sum_{i=1}^2 X_i^{-2} (F^i)^2 \right), \quad (4.68)$$

where

$$V = -4X_1X_2 - 2X_1^{-1}X_2^{-2} - 2X_2^{-1}X_1^{-2} + \frac{1}{2}(X_1X_2)^{-4}, \quad (4.69)$$

$$X_i = e^{-\frac{1}{2}\vec{a}_i \cdot \vec{\varphi}}, \quad (4.70)$$

with

$$\vec{a}_0 = \left(0, -4\sqrt{\frac{2}{5}} \right), \quad \vec{a}_1 = \left(\sqrt{2}, \sqrt{\frac{2}{5}} \right), \quad \vec{a}_2 = \left(-\sqrt{2}, \sqrt{\frac{2}{5}} \right). \quad (4.71)$$

For convenience we also define $A = q_1^2 + q_2^2$ and $B = q_1^2 q_2^2$.

4.3.1 Euclidean action of AdS_7 R-charged black holes

Now we come to the calculation of the difference of Euclidean actions of the R-charged black holes and the pure thermal AdS_7 background spacetime. After Euclidean continuation, this solution becomes

$$ds^2 = (\mathcal{H}_1\mathcal{H}_2)^{-\frac{4}{5}} f d\tau^2 + (\mathcal{H}_1\mathcal{H}_2)^{\frac{1}{5}} (f^{-1} dr^2 + r^2 d\vec{x}^2), \quad (4.72)$$

$$A^i = -i \frac{\sqrt{\mu}(1 - \mathcal{H}_i^{-1})}{4lq_i}. \quad (4.73)$$

And the Euclidean action

$$I_{\text{Euc}} = -\frac{1}{16\pi G_7} \int d^7x \sqrt{g} \left(R - \frac{1}{2}(\partial\vec{\varphi})^2 - \frac{V}{l^2} - \frac{1}{4} \sum_{i=1}^2 X_i^{-2} (F^i)^2 \right). \quad (4.74)$$

To avoid the conical singularity in the Euclidean sector of the black hole solution, the coordinate τ should get a period

$$\beta = \frac{4\pi}{\left((\mathcal{H}_1\mathcal{H}_2)^{-\frac{1}{2}} f'(r) \right) |_{r=r_0}}, \quad (4.75)$$

where r_0 corresponds to the horizon, and is the largest real root of $f(r) = 0$, i.e.,

$$\mu = \frac{r_0^6}{4l^2} \mathcal{H}_1(r_0) \mathcal{H}_2(r_0). \quad (4.76)$$

β is the inverse Hawking temperature of the black hole. The on-shell actions for the black hole and the pure AdS background

$$I_{\text{bulk}}^{bl} = \frac{V(\vec{x})4^5 l^3 \beta}{16\pi G_7} \left[\frac{4}{5} l^4 q_1^2 q_2^2 \left(\frac{1}{r_{uv}^2} - \frac{1}{r_0^2} \right) + \frac{3}{20} l^2 (q_1^2 + q_2^2) (r_{uv}^2 - r_0^2) + \frac{1}{64} (r_{uv}^6 - r_0^6) + \frac{2l^4 \mu}{5} \sum_{i=1,2} \left(\frac{q_i^2}{16l^2 q_i^2 + r_{uv}^4} - \frac{q_i^2}{16l^2 q_i^2 + r_0^4} \right) \right], \quad (4.77)$$

and

$$I_{\text{bulk}}^{ba} = \frac{V(\vec{x})4^5 l^3 \beta'}{16\pi G_7} \left(\frac{1}{64} r_{uv}^6 \right), \quad (4.78)$$

respectively. From equation (4.13), the Euclidean time period of the AdS space is fixed by

$$\beta' = \beta \sqrt{\frac{4l^2 (\mathcal{H}_1\mathcal{H}_2)^{\frac{1}{5}} f}{r^2}} \Big|_{r_{uv}}. \quad (4.79)$$

Furthermore, by explicit calculations one can show that the Gibbons-Hawking surface term and the counterterm discussed in previous section both have no contribution to this action difference. As a result the total action difference is

$$\Delta I = -\frac{V(\vec{x})4^5 l^6 \beta}{16\pi G_7} \left(\frac{\mu}{32l} \right). \quad (4.80)$$

This is always negative, so there is no Hawking-Page phase transition in this case.

When an IR cutoff is introduced, the on-shell action of the black hole becomes

$$I_{bl} = \frac{V(\vec{x})4^5 l^3 \beta}{16\pi G_7} \left[\frac{4}{5} l^4 q_1^2 q_2^2 \left(\frac{1}{r_{uv}^2} - \frac{1}{r_{\max}^2} \right) + \frac{3}{20} l^2 (q_1^2 + q_2^2) (r_{uv}^2 - r_{\max}^2) + \frac{1}{64} (r_{uv}^6 - r_{\max}^6) + \frac{32}{5} l^5 U_H^3 \sum_{i=1,2} \left(\frac{q_i^2}{16l^2 q_i^2 + r_{uv}^4} - \frac{q_i^2}{16l^2 q_i^2 + r_{\max}^4} \right) \right], \quad (4.81)$$

where $r_{\max} = \max[r_0, r_{IR}]$, while for the AdS background, one has

$$I_{ba} = \frac{V(\vec{x})4^5 l^3 \beta'}{16\pi G_7} \left(\frac{1}{64} (r_{uv}^6 - r_{IR}^6) \right). \quad (4.82)$$

Thus the action difference is

$$\Delta I = \frac{V(\vec{x})4^5 l^3 \beta}{16\pi G_7} \left[\frac{\mu l^2}{32} + \frac{1}{64} r_{IR}^6 - \frac{1}{64} r_{\max}^6 - \frac{3}{20} l^2 A r_{\max}^2 - \frac{4Bl^4}{5r_{\max}^2} - \frac{2l^4 \mu}{5} \sum_{i=1}^2 \frac{q_i^2}{r_{\max}^4 + q_i^2 16l^2} \right]. \quad (4.83)$$

If $r_0 < r_{IR}$, one has $r_{\max} = r_{IR}$, and

$$\Delta I = \frac{V(\vec{x})4^5 l^3 \beta}{16\pi G_7} \left[\frac{\mu l^2}{32} - \frac{3l^2 A}{20} r_{IR}^2 - \frac{4Bl^4}{5r_{IR}^2} - \frac{2l^4 \mu}{5} \sum_{i=1}^2 \frac{q_i^2}{r_{IR}^4 + q_i^2 16l^2} \right]. \quad (4.84)$$

When $r_0 > r_{IR}$, we should have $r_{\max} = r_0$, and the action difference becomes

$$\Delta I = \frac{V(\vec{x})4^5 l^3 \beta}{16\pi G_7} \left(\frac{1}{64} r_{IR}^6 - \frac{\mu l^2}{32} \right). \quad (4.85)$$

This action difference will reduce to the case without IR cutoff (4.80) if the cutoff parameter r_{IR} vanishes. It should be noted here, for this R-charged black hole, it is easy to find the counterterm (4.55) does not give any contribution to the Euclidean action. This is different from the cases in 4 and 5 dimensions.

4.3.2 Phase transition with an IR cutoff

We consider the case $r_0 > r_{IR}$. Because $\mu l^2 = r_0^6 \mathcal{H}_1(r_0) \mathcal{H}_2(r_0) / 4$ approaches $r_0^6 / 4$ when r_0 goes to infinity, $\frac{1}{2} r_0^6 - \mu l^2 > 0$ can be easily satisfied for some r_0 big enough. Thus as argued in five dimensional case, for any values of q_i 's, there always exists a value of r_0 which is denoted by $r_{0c}(q_i)$ such that $\frac{1}{2} r_0^6 - \mu l^2 > 0$ if $r_0 > r_{0c}(q_i)$. Therefore, we can always find an IR cutoff $r_{0c}(q_i) < r_{IR} < r_0$ which satisfies $\frac{1}{2} r_{0c}^6 - \mu l^2 > 0$. This means that introducing a proper IR cutoff can lead to a confinement phase. The deconfinement transition happens when the action difference (4.85) changes its sign.

In figure 15 and 17 we plot the $r_0 - q$ phase diagrams for the case of $q_1 = q, q_2 = 0$ and $q_1 = q_2 = q$, respectively. The five solid curves correspond to the phase transition curves, and each curve has a fixed IR cutoff r_{IR} . With the color changing from black to

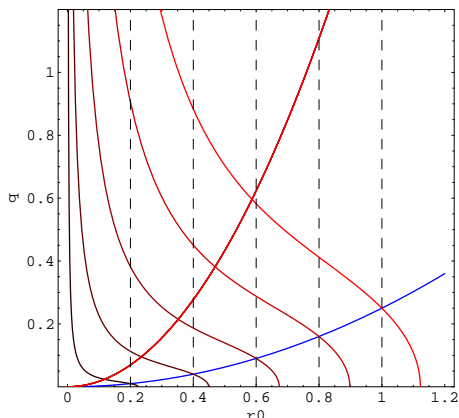


Figure 15: $r_0 - q$ phase diagram of 7-dimensional R-charged black hole with $q_1 = q, q_2 = 0$.

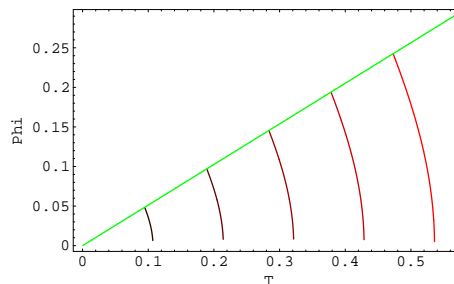


Figure 16: $T - \phi$ phase diagram of 7-dimensional R-charged black hole with $q_1 = q, q_2 = 0$.

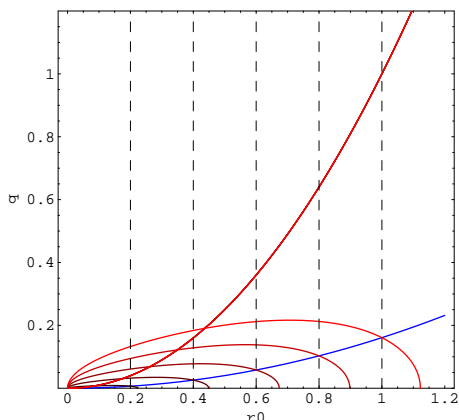


Figure 17: $r_0 - q$ phase diagram of 7-dimensional R-charged black hole with $q_1 = q_2 = q$.

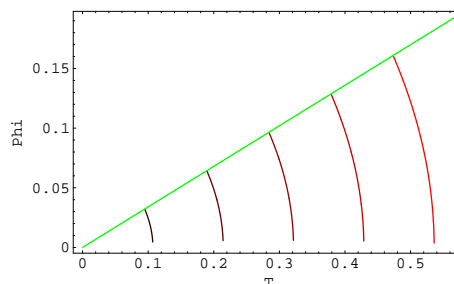


Figure 18: $T - \phi$ phase diagram of 7-dimensional R-charged black hole with $q_1 = q_2 = q$.

red, the values of r_{IR} increase from 0.2 to 1.0 with a step 0.2. The dash curves represent $r_0 = 0.2, 0.4, 0.6, 0.8$ and 1.0, respectively. The blue curves stand for the requirement of $r_0 > r_{IR}$. In these figures, the red curves starting from the origin correspond to $q^2 = 3r_0^4$ and $q^2 = r_0^4$, respectively. They are local thermodynamic stability curves, determined by the Hessian of the Euclidean action with respect to r_0 and q_i with β and Φ_i fixed. Thus only the regions below these blue curves satisfy the condition $r_0 > r_{IR}$. As a result, The thermodynamics is always local stable in those regions.

Figures 16 and 18 give the $T - \phi$ phase diagrams for the case of $\Phi_1 = \phi, \Phi_2 = 0$ and $\Phi_1 = \Phi_2 = \phi$, respectively. The green curves correspond to the requirement $r_{IR} < r_0$. With the colors changing from black to blue, the value of r_{IR} increases from 0.2 to 1.0 with a step 0.2. Again we only give the regions where the deconfinement transitions happen.

5. Conclusion

In this paper we have studied in grand canonical ensemble the Hawking-Page phase transition associated with decoupling limits of black D p -branes ($0 \leq p \leq 4$) and R-charged AdS_5 , AdS_4 and AdS_7 black holes coming from spherical reduction of rotating black D3-, M2- and M5-branes respectively. The Hawking-Page phase transition can be identified with the confinement-deconfinement phase transition of dual SYM theories at finite temperature.

For the case of the near horizon geometries of black D p -branes, there does not exist any phase transition for the dual SYM theories in non-compact spacetime $S^1 \times R^p$, although when $p \neq 3$, the dual theories are not conformal. The Euclidean action difference between the near horizon geometries of black D p -branes and BPS D p -branes are always negative, which means that the dual field theories are always in the deconfinement phase. When we introduce an IR cutoff, as the case of hard-wall AdS/QCD model, a confinement phase can be realized. And then the deconfinement transition for the dual SYM theories occurs at some critical temperature which is determined by the IR cutoff.

The Hawking-Page phase transition also does not appear for the R-charged AdS_5 , AdS_4 , and AdS_7 black holes with Ricci flat horizon. These black holes are dual to some R-charged supersymmetric field theories on the AdS boundary. When we introduce a proper IR cutoff, once again, we can realize the deconfinement phase transitions for those field theories. We have analyzed in some detail the phase diagrams associated with those R-charged black holes.

Acknowledgments

LMC and YWS would like to thank Bin Hu, Ding Ma and Wei-Shui Xu for useful discussions and kind help. This work was supported partially by grants from NFSC, China (No. 10325525 and No.90403029), and a grant from the Chinese Academy of Sciences.

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